Statistical modelling

#1.a Hypothesis tests

Dr. Léo Belzile
HEC Montréal

Hypothesis testing

Trial analogy

Screenshot of the courtoom drama *Twelve Angry Men* (1957)

Recipe

An hypothesis test is a binary decision rule

Below are the different steps to undertake:

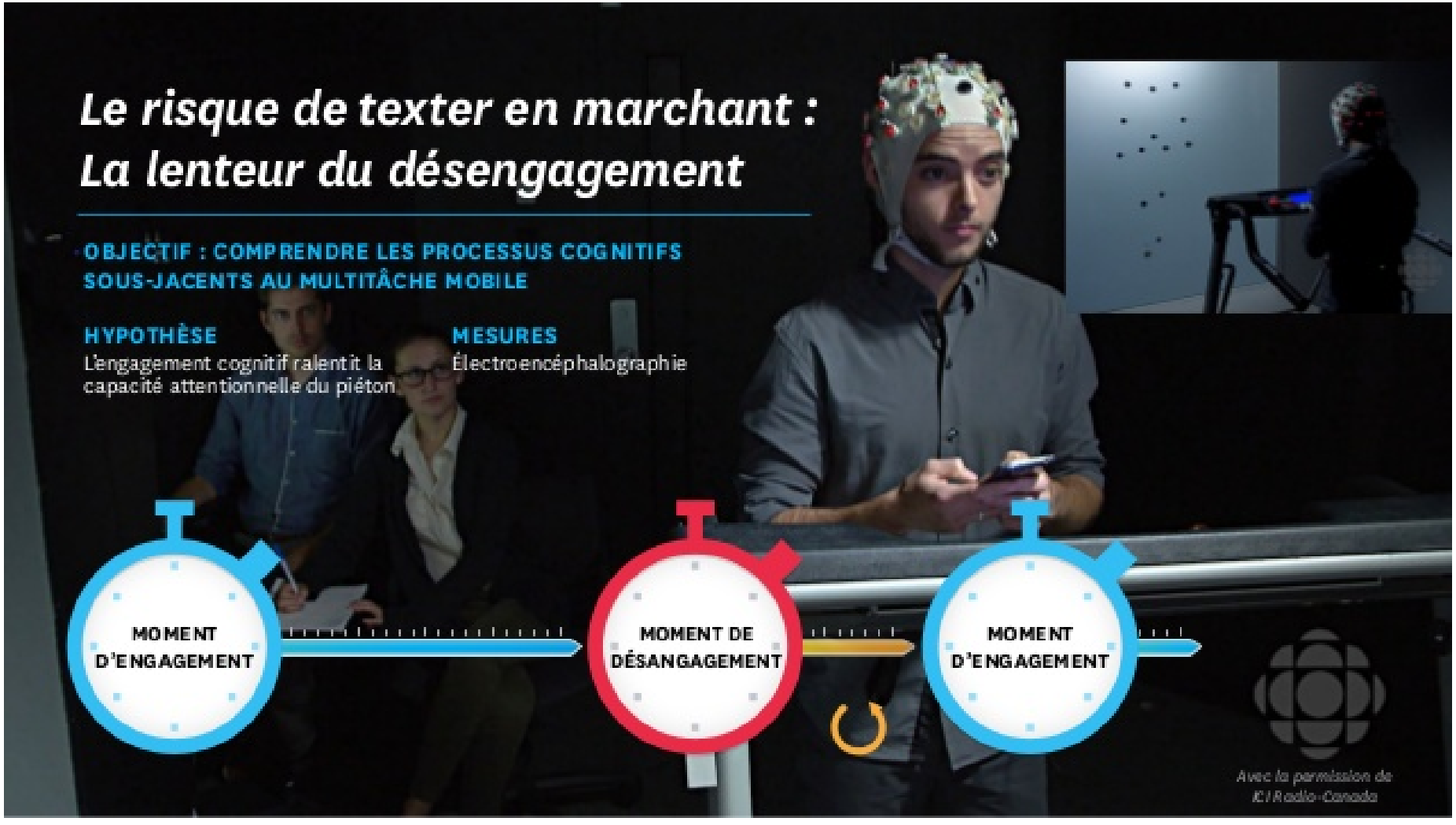
1. Define the variables of interest
2. Formulate the alternative and the null hypotheses, H_a and H_0
3. Choose the test statistic and compute the latter on the sample
4. Compare the numerical value with the null distribution
5. Obtain the *p*-value
6. Conclude in the setting of the problem

Tech3Lab

Le Tech3Lab est le plus important laboratoire en expérience utilisateur (UX) en Amérique du Nord

Le Tech3Lab est un laboratoire de recherche appliquée en sciences de la gestion qui se spécialise dans l'analyse des interactions entre les interfaces technologiques des organisations et leurs employés ou consommateurs

Texting while walking: a dangerous habit?



Study details

- 35 participants took part in the study.
- Each person had to walk on a treadmill in front of a screen where obstacles were projected.
- In one of the sessions, the subjects walked while talking on a cell phone, whereas in another session, they walked while texting.
- The order of these sessions was determined *at random*.
- Different obstacles were randomly projected during the session.
- We are only interested in one kind of scenario: a cyclist riding towards the participant.

Characteristics

- Population: adults
- Sample: 35 individuals
- Variables:
  - Time to perceive an obstacle: quantitative
  - Distraction type (cellphone call or texting): nominal variable
- Variable of interest: time (in seconds) that it takes for a person to notice the obstacle when walking while texting or talking on a cell phone (measure through an encephalogram)

#1. Define the variables of interest

- $\mu_{\texttt{c}}$ be the average reaction time (in seconds) during a call (`c`)
- $\mu_{\texttt{t}}$ be the average reaction time (in seconds) while texting (`t`)

#2. Formulate the null and alternative hypothesis

- Hypothesis of interest: does texting increases distraction?
  - $\mathscr{H}_a : \mu_{\texttt{t}} > \mu_{\texttt{c}}$ (one-sided)
- Null hypothesis (Devil's advocate)
  - $\mathscr{H}_0 : \mu_{\texttt{t}} \leq \mu_{\texttt{c}}$

Express the hypothesis in terms of the difference of means

$$\mathscr{H}_a : \mu_{\texttt{t}} - \mu_{\texttt{c}} > 0.$$

#3.Choose the test statistic

We compare the difference of the mean reaction time

- one-sample t-test for $\texttt{t} - \texttt{c}$ (paired t-test)

$$T_D = \frac{\overline{D} - \mu_0}{\text{se}(\overline{D})}$$

- $\overline{D}$ is the mean difference in the sample.
- Under $\mathscr{H}_0 : \mu_0 = \mu_{\texttt{t}} - \mu_{\texttt{c}} = 0$.
- The standard error of $\overline{D}$ is $\text{se}(\overline{D}) = S_D/\sqrt{n}$, where S_D is the standard deviation of the variables D_i and n the sample size.

- SAS code ✚ SAS output ✚ R code ✚ R output

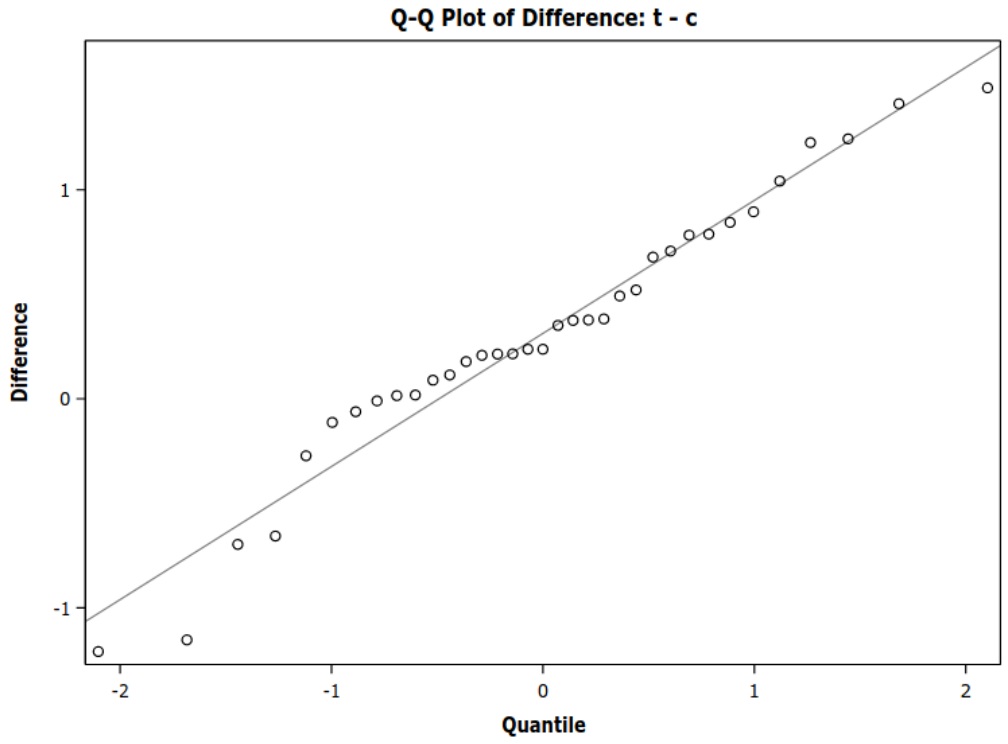
```
proc ttest data=statmod.distraction side=u;
paired t*c;
run;
```

Difference: t - c

N	Mean	Std Dev	Std Err	Minimum	Maximum
35	0.3130	0.6369	0.1077	-1.2100	1.4880

Mean	95% CL Mean		Std Dev	95% CL Std Dev	
0.3130	0.1309	Infty	0.6369	0.5152	0.8345

DF	t Value	Pr > t
34	2.91	0.0032



```
data(distraction, package = "hecstatmod")
with(distraction,
     t.test(t-c, alternative = "greater", mu = 0)
     )
```

```
##
##     One Sample t-test
##
## data:  t - c
## t = 2.91, df = 34, p-value = 0.0032
## alternative hypothesis: true mean is greater than 0
## 95 percent confidence interval:
##  0.13092     Inf
## sample estimates:
## mean of x
##   0.31297
```

#4. Compare the statistic with the null distribution

The null distribution is Student-t with 34 degrees of freedom, St_{34}.

We are only interested in the probability that $T_D > 2.91$ under $\mathscr{H}_0$.

#5. Obtain a p-value

The p-value is 0.0032, which is smaller than $\alpha = 5\%$.

#6. Conclude in the setting of the problem

We reject $\mathscr{H}_0$, meaning that the reaction time is significantly higher when texting than talking on the cellphone while walking.

The estimated mean difference is 0.313 seconds.